Graph Theory
Class-BCA IV Semester

Dr. Vijay Kant Sharma

Assistant professor ,

Department Of Computer Application

Jagatpur P.G. College, Varanasi

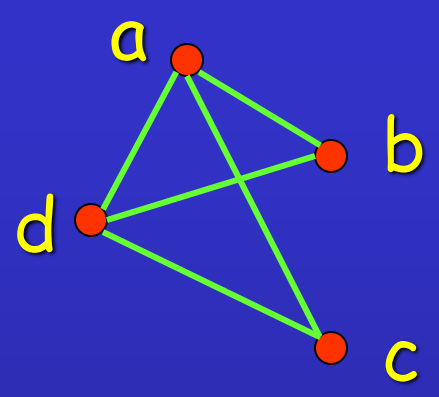
Affiliated to Mahatma Gandhi Kashi vidhyapith Varanasi

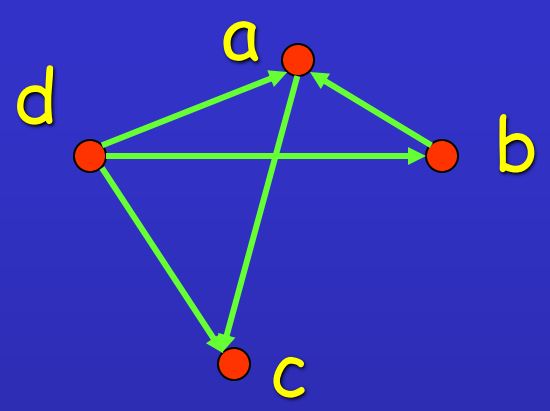
Email-mzp.vijay@gmail.com

OUTLINE-UNIT-II

Representing Graphs and Graph Isomorphism

Representing Graphs





Vertex	Adjacent Vertices
a	b, c, d
b	a, d
c	a, d
d	a, b, c

Initial Vertex	Terminal Vertices
a	c
b	a
c	
d	a, b, c

Representing Graphs

Definition: Let $G = (V, E)$ be a simple graph with $|V| = n$. Suppose that the vertices of G are listed in arbitrary order as $v_1, v_2, \ldots, v_n$.

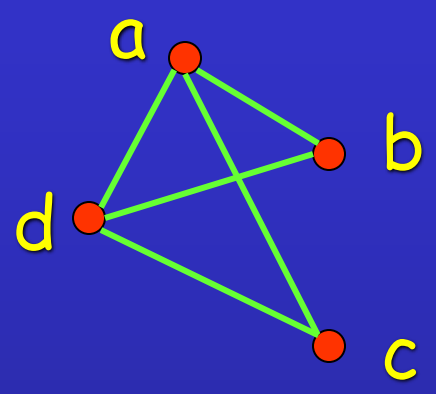
The **adjacency matrix** A (or A_G) of G, with respect to this listing of the vertices, is the $n \times n$ zero-one matrix with 1 as its (i, j) entry when v_i and v_j are adjacent, and 0 otherwise.

In other words, for an adjacency matrix $A = [a_{ij}]$,

$a_{ij} = 1$ if $\{v_i, v_j\}$ is an edge of G,
$a_{ij} = 0$ otherwise.

Representing Graphs

Example: What is the adjacency matrix A_G for the following graph G based on the order of vertices a, b, c, d ?



Solution:

$$A_G = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Note: Adjacency matrices of undirected graphs are always symmetric.

Representing Graphs

Definition: Let $G = (V, E)$ be an undirected graph with $|V| = n$. Suppose that the vertices and edges of G are listed in arbitrary order as $v_1, v_2, \ldots, v_n$ and $e_1, e_2, \ldots, e_m$, respectively.

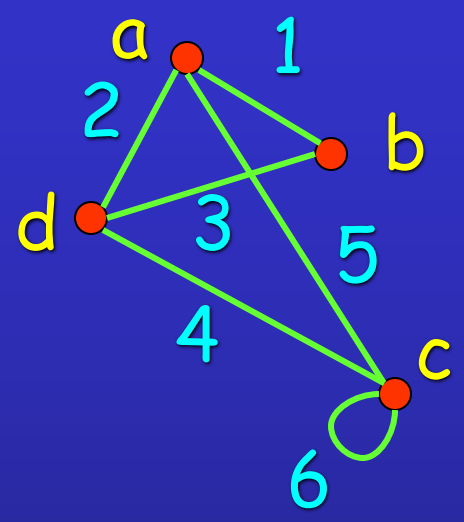
The **incidence matrix** of G with respect to this listing of the vertices and edges is the $n \times m$ zero-one matrix with 1 as its (i, j) entry when edge e_j is incident with v_i, and 0 otherwise.

In other words, for an incidence matrix $M = [m_{ij}]$,

$m_{ij} = 1$ if edge e_j is incident with v_i
$m_{ij} = 0$ otherwise.

Representing Graphs

Example: What is the incidence matrix M for the following graph G based on the order of vertices a, b, c, d and edges 1, 2, 3, 4, 5, 6?



Solution:

$$M = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Note: Incidence matrices of directed graphs contain two 1s per column for edges connecting two vertices and one 1 per column for loops.

Isomorphism of Graphs

Definition: The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are **isomorphic** if there is a bijection (an one-to-one and onto function) f from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if f(a) and f(b) are adjacent in G_2, for all a and b in V_1.

Such a function f is called an **isomorphism**.

In other words, G_1 and G_2 are isomorphic if their vertices can be ordered in such a way that the adjacency matrices M_{G_1} and M_{G_2} are identical.

Isomorphism of Graphs

From a visual standpoint, G_1 and G_2 are isomorphic if they can be arranged in such a way that their **displays are identical** (of course without changing adjacency).

Unfortunately, for two simple graphs, each with n vertices, there are **n! possible isomorphisms** that we have to check in order to show that these graphs are isomorphic.

However, showing that two graphs are **not** isomorphic can be easy.

Isomorphism of Graphs

For this purpose we can check **invariants**, that is, properties that two isomorphic simple graphs must both have.

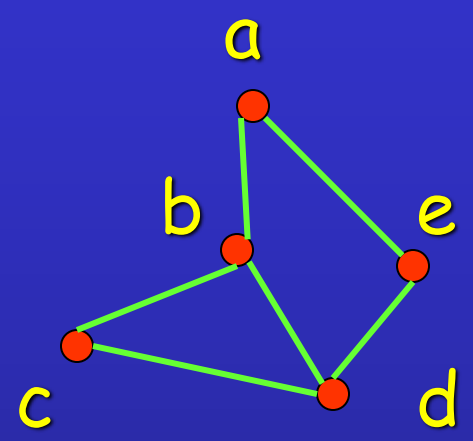
For example, they must have

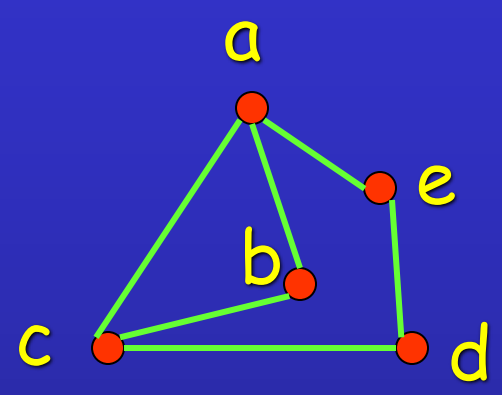
- the same number of vertices,
- the same number of edges, and
- the same degrees of vertices.

Note that two graphs that **differ** in any of these invariants are not isomorphic, but two graphs that **match** in all of them are not necessarily isomorphic.

Isomorphism of Graphs

Example I: Are the following two graphs isomorphic?

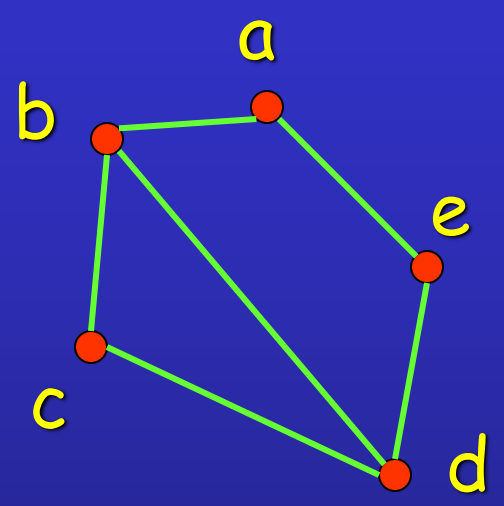


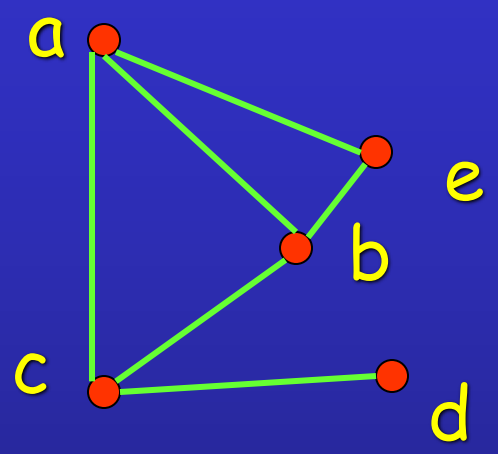


Solution: Yes, they are isomorphic, because they can be arranged to look identical. You can see this if in the right graph you move vertex b to the left of the edge {a, c}. Then the isomorphism f from the left to the right graph is: f(a) = e, f(b) = a, f(c) = b, f(d) = c, f(e) = d.

Isomorphism of Graphs

Example II: How about these two graphs?



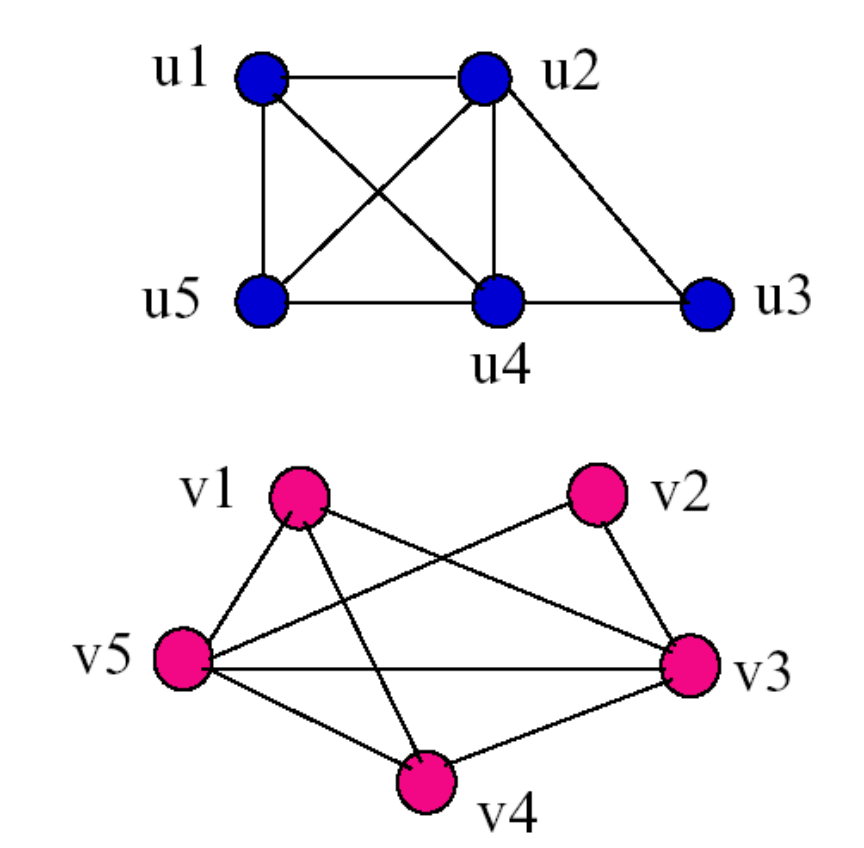


Solution: No, they are not isomorphic, because they differ in the degrees of their vertices.

Vertex d in right graph is of degree one, but there is no such vertex in the left graph.

Examples

Determine if the following two graphs G_1 and G_2 are isomorphic:



Question 35, p. 619, and 41, p. 620

QUESTIONS:-

1. What Isomorphism in graph theory?
2. Explain two examples of Isomorphism of graph?
3. What is 1 Isomorphism and 2 Isomorphism?

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Dr.Vijay kant Sharma

Assistant professor

Department of computer Application
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Thanks