COMPUTER ARCHITECTURE & ASSEMBLY LANGUAGE
BACHELOR OF COMPUTER APPLICATIONS (B.C.A)
III Semester

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UNIT - III

Computer Arithmetic

General concept

- Decimal addition

```
(carry)  1_
         19
       + 7
         26
```

- Binary addition

```
( carry)  111_
          10011
        + 111
          11010
```

- `16+8+2 = 26`

Signed and unsigned additions

- Unsigned addition in 4-bit arithmetic

```
( carry)   11_
           1011
         + 0011
           1110
```

- `11 + 3 = 14`
  `(8 + 4 + 2)`

- Signed addition in 4-bit arithmetic

```
( carry)   11_
           1011
         + 0011
           1110
```

- `-5 + 3 = -2`

Signed and unsigned additions

- ***Same rules*** apply even though bit strings represent ***different values***
- Sole difference is ***overflow handling***

Overflow handling (I)

- No overflow in signed arithmetic

```
( carry)  111_
          1110
        + 0011
        ------
          0001
```

- `-2 + 3 = 1 (correct)`

- Signed addition in 4-bit arithmetic

```
( carry)  1___
          0110
        + 0011
        ------
          1001
```

- `6 + 3 ≠ -7 (false)`

Overflow handling (II)

- In signed arithmetic an overflow happens when
  - The ***sum of two positive numbers*** exceeds the maximum positive value that can be represented using n bits: $2^{n-1} - 1$
  - The s***um of two negative numbers*** falls below the minimum negative value that can be represented using n bits: -2^{n-1}

Example

- Four-bit arithmetic:
  - Sixteen possible values
  - Positive overflow happens when result > 7
  - Negative overflow happens when result < -8
- Eight-bit arithmetic:
  - 256 possible values
  - Positive overflow happens when result > 127
  - Negative overflow happens when result < -128

An interesting consequence

- Most C compilers ignore overflows
  - C compilers must use unsigned arithmetic for their integer operations
- Fortran compilers expect overflow conditions to be detected
  - Fortran compilers must use signed arithmetic for their integer operations

Negating a number

- Toggle all bits then add one

In 4-bit arithmetic (I)

0000	0	1111	+1 = 0000	0
0001	1	1110	+1 = 1111	-1
0010	2	1101	+1 = 1110	-2
0011	3	1100	+1 = 1101	-3
0100	4	1011	+1 = 1100	-4
0101	5	1010	+1 = 1011	-5
0110	6	1001	+1 = 1010	-6
0111	7	1000	+1 = 1001	-7

Decimal multiplication

```
(carry) 1_
          37
       x  12
       ----
          74
         370
       ----
         444
```

- What are the rules?
  - Successively multiply the multiplicand by each digit of the multiplier starting at the right shifting the result left by an extra left position each time each time but the first
  - Sum all partial results

Binary multiplication

```
(carry)
 111
   1101
  x 101
  ------
   1101
     00
 110100
-------
1000001
```

- What are the rules?
  - Successively multiply the multiplicand by each digit of the multiplier starting at the right shifting the result left by an extra left position each time each time but the first
  - Sum all partial results
- **Binary multiplication is easy!**

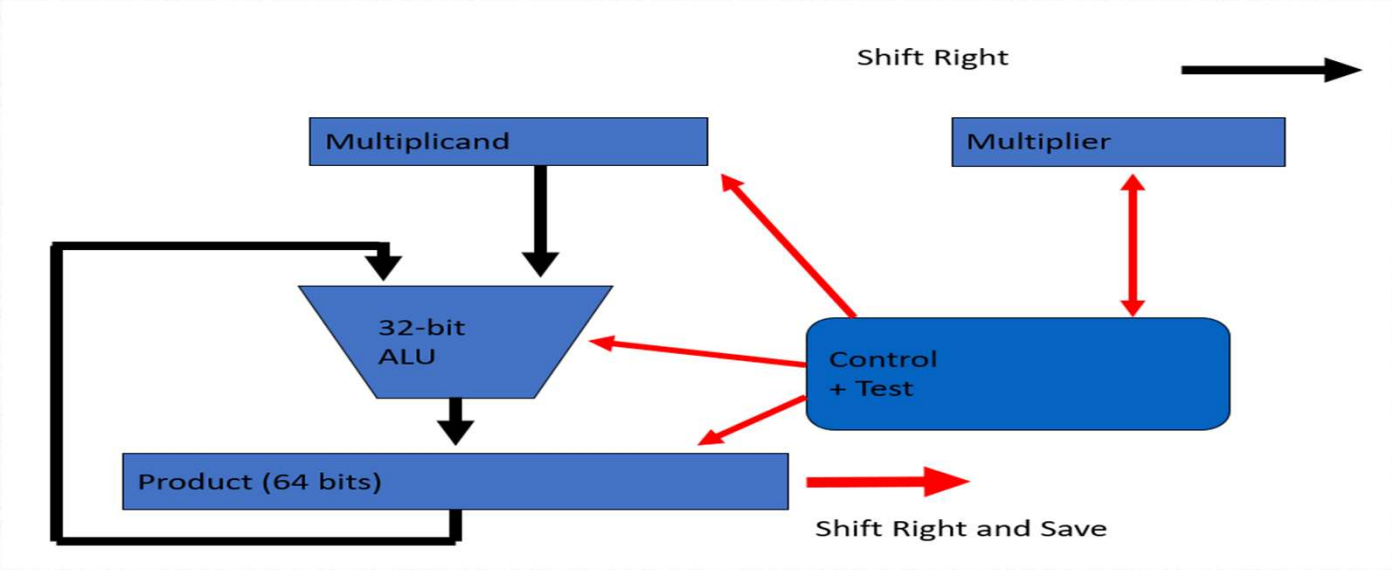
Binary multiplication table

X	0	1
0	**0**	**0**
1	**0**	**1**

Algorithm

- Clear contents of 64-bit product register
- For (i = 0; i <32; i++) {
  - If (LSB of multiplier_register ==1)
    - Add contents of multiplicand register to product register
  - Save LSB of product register
  - Shift ***right*** one position ***both*** multiplier register and product register
- } / / for loop

Multiplier: Second version



Example (I)

- Multiply **0011** by **0011**
- **<u>Start</u>**

Multiplicand	Multiplier	Product	Result
0011	**0011**	**--**	**--**

- **<u>First bit</u>**

Multiplicand	Multiplier	Product	Result
0011	**001<u>1</u>**	**0011**	**--**

Decimal division (long division)

```
   303
7 | 2126
  -210
     26
    -21
      5
```

- What are the rules?
  - Repeatedly try to subtract smaller multiple of divisor from dividend
  - Record multiple (or zero)
  - At each step, repeat with a lower power of ten
  - Stop when remainder is smaller than divisor

Binary division

```
        011
11 | 1011
     x
    -11
     1011
    >-11
x     101
    >>-11
      10
```

- What are the rules?
  - Repeatedly try to subtract powers of two of divisor from dividend
  - Mark 1 for success, 0 for failure
  - At each step, shift divisor one position to the right
  - Stop when remainder is smaller than divisor

Division Algorithm

- **For i in range(0,33) :** # from 0 to 32
  - *Subtract* contents of ***divisor register*** from ***remainder register***
  - **If remainder ≥ 0 :**
    - Shift ***quotient register to the left***
    - Set new rightmost bit to **1**

    Else :
    - ***Undo*** subtraction
    - Shift ***quotient register to the left***
    - Set new rightmost bit to **0**
  - Shift ***right*** one position contents of divisor register

FLOATING POINT OPERATIONS

- Used to represent ***real numbers***
- Very similar to ***scientific notation***

$$3.5\times10^{6},\ 0.82\times10^{-5},\ 75\times10^{6},\ \ldots$$

- Both decimal numbers in scientific notation and floating point numbers can be normalized:

$$3.5\times10^{6},\ 8.2\times10^{-6},\ 7.5\times10^{7},\ \ldots$$

Example

- Sign bit is zero:
  Number is positive
- Biased exponent is 127
  Power of two is zero
- Normalized binary value is
  1.0000000
- Number is **$1 \times 2^0 = 1$**

0	01…1	00000000000000000000000000000

Decimal floating point addition

- **$9.25 \times 10^3 + 8.22 \times 10^2 = ?$**
- Denormalize number with smaller exponent:
  $9.25 \times 10^3 + 0.822 \times 10^3$
- Add the numbers:
  $9.25 \times 10^3 + 0.822 \times 10^3 = 10.072 \times 10^3$
- Normalize the result:
  $10.072 \times 10^3 = 1.0072 \times 10^4$

Binary floating point addition

- Say **101 + 11** or $\mathbf{1.01 \times 2^2 + 1.1 \times 2^1}$
- Denormalize number with smaller exponent:
  $\mathbf{1.01 \times 2^2 + 0.11 \times 2^2}$
- Add the numbers:
  $\mathbf{1.01 \times 2^2 + 0.11 \times 2^2 = 10.00 \times 2^2}$
- Normalize the results
  $\mathbf{10.00 \times 2^2 = 1.000 \times 2^3}$

Excercises

- *How would you represent* 0.5 *in double precision?*
- *How would you convert this double-precision value into a single precision format?*
- *When doing accounting, we could do all the computations in cents using integer arithmetic. What would we win? What would we lose?*

Reference

- Reference Books:
- 1. Leventhal, L.A, “Introduction to Microprocessors”, Prentice Hall of India
- 2. Mathur, A.P., “Introduction to Microprocessors”, Tata McGraw Hill
- 3. Rao, P.V.S., “Prospective in Computer Architecture” , Prentice Hall of India

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THANK YOU