Numerical Methods

Class- BCA Vth Semester

Topic: Gauss Seidel Method

Dr. Dharm Raj Singh

Assistant Professor, (HOD)

Department of Computer Application

Jagatpur P. G. College, Varanasi

Affiliated to Mahatma Gandhi Kashi Vidhyapith Varanasi

Email- dharmrajsingh67@yahoo.com

Outline

UNIT-IV: Solution of Linear Equation

- Gauss Seidel Method
- LU Decomposition /Crout's Method
  Finding the [U] matrix
- Finding the [L] matrix

Gauss Seidel Method

Algorithm for Gauss-Seidel Method

Given system is:

$a_1x+b_1y+c_1z=d_1$

$a_2x+b_2y+c_2z=d_2$

$a_3x+b_3y+c_3z=d_3$

Convert the 1st equation in terms of 1st variable, 2nd equation in terms of 2nd variable and so on.

$x=(d_1-b_1y-c_1z)/a_1$

$y=(d_2-c_2z-a_2x)/b_2$

$z=(d_3-a_3x-b_3y)/c_3$

Assume initial guesses as x_0,y_0,z_0.

Substituting x_0,y_0,z_0, find x_1, y_1, z_1 from above converted form as

$x_1=(d_1-b_1y_0-c_1z_0)/a_1$

$y_1=(d_2-c_2z_0-a_2x_1)/b_2$

$z_1=(d_3-a_3x_1-b_3y_1)/c_3$

- **If $|x_0-x_1|<accuracy$ && $|y_0-y_1|<accuracy$&& $|z_0-z_1|<accuracy$, required roots are x_1,y_1,z_1, Else the values of x_i, y_i, z_i for the consecutive calculations remain the same.**

LU Decomposition /Crout’s Method

Method

For most non-singular matrix [A] that one could conduct Naïve Gauss Elimination forward elimination steps, one can always write it as

$[A] = [L][U]$

where

$[L]$ = lower triangular matrix

$[U]$ = upper triangular matrix

Method: [A] Decompose to [L] and [U]

$$[A] = [L][U] = \begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

[*U*] is the same as the coefficient matrix at the end of the forward elimination step.

[*L*] is obtained using the *multipliers* that were used in the forward elimination process

Finding the [*U*] matrix

Using the Forward Elimination Procedure of Gauss Elimination

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

Step 1:

$$\frac{64}{25} = 2.56; \quad Row2 - Row1(2.56) = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 144 & 12 & 1 \end{bmatrix}$$

$$\frac{144}{25} = 5.76; \quad Row3 - Row1(5.76) = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{bmatrix}$$

Finding the [U] Matrix

Matrix after Step 1:
$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{bmatrix}$$

Step 2:
$$\frac{-16.8}{-4.8} = 3.5; \quad Row3 - Row2(3.5) = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

$$[U] = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

Finding the [L] matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}$$

Using the multipliers used during the Forward Elimination Procedure

From the first step of forward elimination

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

$$\ell_{21} = \frac{a_{21}}{a_{11}} = \frac{64}{25} = 2.56$$

$$\ell_{31} = \frac{a_{31}}{a_{11}} = \frac{144}{25} = 5.76$$

Exercise

2. Solve the following system of equations using Gauss-Elimination method:

(a)
$$x - y + z = 1$$
$$-3x + 2y - 3z = -6$$
$$2x - 5y + 4z = 5$$
[Ans. −2, 3, 6]

(b)
$$x + 3y + 6z = 2$$
$$x - 4y + 2z = 7$$
$$3x - y + 4z = 9$$
[Ans. 2, −1, $\frac{1}{2}$]

(c)
$$5x + y + z + u = 4$$
$$z + 7y + z + u = 12$$
$$x + y + 6z + u = -5$$
$$x + y + z + u = -6$$
[Ans. 1, 2, −1, −2]

3. What do you understand by ill-conditioned equations? Consider the following system of equations:

$$100x - 200y = 100$$
$$-200x + 401y = 100$$

Determine, whether given system is ill-conditioned or not.

4. Solve the following system of equations by Jacobi's iterations method:

(a)
$$2x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$
[Ans. 1, −1, 1]

(b)
$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$
[Ans. 1.08, 1.95, 3.16]

5. Using Gauss-Seidel method, solve the following system of equations:

(a)
$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$2x + 2y + 10z = 14$$
[Ans. 1, 1, 1]

(b)
$$2x - y + z = 5$$
$$2 + 3y - 2z = 7$$
$$x + 2y + 3z = 10$$
[Ans. 3, 2, 1]

(c)
$$20x + y - 2z = -17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$
[Ans. 1, −1, 1]

References

- Jain and Iyenger, Numerical Methods for Scientific and Engineering Computation, New Age International Limited Publication, 2007.
- Stevan C. Chapra, Applied Numerical Methods with MATLAB, Tata Mcgraw-Hill, 2007.
- E. Balagurusamy, Numerical Methods, Tata Mcgraw-Hill, 2007
- S.S. Sastry, Introductory Methods of Numerical Analysis, 5^{th} ed. PHI Pvt. Ltd. New Delhi ,2012.
- Scarbourogh, “Numerical Analysis”.
- Gupta & Bose S.C. “Introduction to Numerical Analysis, “Academic Press, Kolkata, 3. S.S.Shashtri, “Numerical Analysis”, PHI
- Mathews, John H. Numerical methods for mathematics, science and engineering. Vol. 10. Prentice-Hall international, 1992.
- Ferziger, Joel H. Numerical methods for engineering application. Vol. 1. New York: Wiley, 1981.
- Rao, Singiresu S. Applied numerical methods for engineers and scientists. Prentice Hall Professional Technical Reference, 2001.

Declaration

“The content is exclusively meant for academic purpose and for enhancing teaching and learning. Any other use for economic/commercial purpose is strictly prohibited. The users of the content shall not distribute, disseminate or share it with anyone else and its use is restricted to advancement of individual knowledge. The information provided in this e-content is authentic and best as per knowledge”.

Dr. Dharm Raj Singh
Assistant Professor, (HOD)
Department of Computer Application
JagatpurP. G. College, Varanasi

Thanks